Electrically charged compact stars and formation of charged black holes

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We study the effect of electric charge in compact stars assuming that the charge distribution is proportional to the mass density. The pressure and the density of the matter inside the stars are large, and the gravitational field is intense. This indicates that electric charge and a strong electric field can also be present. The relativistic hydrostatic equilibrium equation, i.e., the Tolman-Oppenheimer-Volkoff equation, is modified in order to include electric charge. We perform a detailed numerical study of the effect of electric charge using a polytropic equation of state. We conclude that in order to see any appreciable effect on the phenomenology of the compact stars, the electric fields have to be huge ($\sim 10^{21}$ V/m), which implies that the total charge is $Q \sim 10^{20}$ Coulomb. From the local effect of the forces experienced on a single charged particle, it is expected that each individual charged particle is quickly ejected from the star. This in turn produces a huge force imbalance, and the gravitational force overwhelms the repulsive Coulomb and fluid pressure forces. The star can then collapse to form a charged black hole before all the charge leaves the system.

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I. INTRODUCTION

The study of the effect of electric charge and electric field in a gravitationally bound system has been done previously by some authors. Rosseland, in 1924 [1] (see also Eddington [2]), studied the possibility that a self gravitating star on Eddington's theory, where the star is modeled by a ball of hot ionized gas, did contain a net charge. In such a system the electrons (lighter particles) tend to rise to the top because of the difference in the partial pressure of electrons compared to that of ions (heavier particles). The motion of electrons to the top and further escape from the star is stopped by the electric field created by the charge separation. The equilibrium is attained after some amount of electrons escape leaving behind an electrified star whose net positive charge is of about 100 Coulomb per solar mass, and building an interstellar gas with a net negative charge. As shown by Bally and Harrison [3], this result applies to any bound system whose size is smaller than the Debye length of the surrounding media. In fact, one should expect a star like the sun to hold some amount of net charge due to the much more frequent escape of elec-

trons than that of protons. Moreover, one should also expect that the escape would stop when the electrostatic energy of an electron $e\Phi$ is of the order of its thermal energy kT. This gives for a ball of hot matter with the sun radius, a net charge $Q \sim 6.7 \times 10^{-6} T$ (in Coulomb). Hence, the escape effect cannot lead to a large net electric charge, and the conclusion is that a star formed by an initially neutral gas cannot acquire a net electric charge larger than about 100C per solar mass.

In the case of cold stars, the gravitational pull is balanced by the degeneracy pressure of the particles. One can ask whether a net charge can have any effect on the structure of the system. For a star of mass M and charge Q, the electrostatic energy of a particle at radius r, eQ/r, is balanced by its gravitational energy, mM/r, where e and m are the charge and mass of the particle, say a proton. For a spherical ball this gives a charge of approximately 100C per solar masses (see Glendenning [4]). This is clearly so for Newtonian stars. For compact stars, the high density and relativistic effects must be taken into account in order to reproduce with precision, the phenomenologies such as the mass and the radius. These effects also reflect, in principle, in the allowed net charge of a compact star, the star can take some more charge to be in equilibrium (see e.g. Bekenstein [5]).

That the observed stars, like the Sun or a neutron star, in equilibrium cannot support a great amount of charge comes from the fact that the particles that compose a star have a huge charge to mass ratio, as is the case for a proton or an electron. For highly relativistic stars, full general relativity is necessary,

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and the situation might be different. One can have highly compact stars, whose radius is on the verge of forming an event horizon, such that the huge gravitational pull can be balanced by huge amounts of net charge. This type of configurations were raised by Bekenstein [5] and further developed in the studies by Zhang et al. [6], de Felice and Yu [7], de Felice et al. [8], Yu and Liu [9], Anninos and Rothman [10] and others. For instance, Zhang et al. [6] found that the structure of a neutron star, for a degenerate relativistic fermi gas, is significantly affected by the electric charge just when the charge density is close to the mass density (in geometric units). In the investigations by de Felice et al., and by Anninos and Rothman, they assumed that the charge distribution followed particular functions of the radial coordinate.

In passing, we can mention that if the charge to mass ratio of the particles that make the star is low, say one or of order one, then a star can contain a huge amount of charge. For instance, one could think of dust particles containing a large quantity of neutral particles, say 10^{18} neutrons, and one proton. This would give a charge to mass ratio of the order one. The dust particles could then cluster around each other and form a star. Theoretically these stars have been considered. The Newtonian theory of gravitation and electrostatics admit equilibrium configurations of charged fluids where the charge density $\rho_{\rm ch}$ can be as large as the mass density ρ, in appropriate units. For instance, a static continuous distribution of charged dust matter with zero pressure will be in equilibrium everywhere if $\rho_{\rm ch} = \pm\rho$ (in geometric units). This gives for a ball of dust with the mass of the sun a net charge of about 1.7×10^{20} Coulomb. The general relativistic analog for charged dust stars was discovered by Majumdar [11] and by Papapetrou [12], and further discussed by Bonnor [13] and several other authors (see [14] for a review). We will not pursue this line here.

In this paper we shall take the ideas initiated in [5] and continue in [6, 7, 8, 9, 10], and focus attention on the effect of charge in cold compact stars, made of neutrons, protons and electrons, say. We assume that the charge density goes with mass density and write $\rho_{\rm ch} = \alpha\rho$, α being a positive (negative) constant for a positive (negative) charge density. We further assume that the net charge in the system is in the form of trapped charged particles carrying positive electric charge. Later we will show that the negative charge will also give the same results, thus the sign of the charge has no effects. From the Einstein-Maxwell equations, we write a modified Tolman-Oppenheimer-Volkoff (TOV) equation (see [5]) where the energy density which appears from the electrostatic field will add up to the total energy density of the system, which in turn will help in the gaining of the total mass of the system. The fundamental difference to the standard relativistic TOV equation from the uncharged case is the presence of the Maxwell stress tensor, which in the modified TOV equation contributes to the energy density, and to the pressure, besides the presence of a potential gradient term from the coulombian term. We solve the modified TOV equation for polytropic equations of state assuming that the charge density goes with the matter density and discuss the results. We will show that a previous estimate by Bekenstein is correct [5]: in order to see any appreciable effect on the phenomenology of the neutron stars, the charge and the electrical fields have to be huge, the large electric field being able to start producing pair creation of particles.

Furthermore, we find solutions with huge amounts of charge. How this extra charge is formed in the star is not the concern here. A mechanism to generate charge asymmetry for charged black holes has been suggested recently [15] and the same may be applied for compact stars too. We are not claiming that compact stars always have so large a charge and strong electric field. Indeed, one has to raise the question what happens to a single charged particle inside the system. It will face a huge electrostatic force and will soon escape the system making the star unstable. During that process, the gravitational force, which was previously balanced to some extent by the repulsive Coulombian force, will overpower the inside pressure and the system will collapse to form a black hole. However, not all the charge has time to escape from the system and hence the charge will remain trapped inside the event horizon to form a charged black hole. This scenario could be applicable for newly born charged compact stars and their rapid transformation to charged black holes.

This paper is organized in the following sections. In section II, we give the general relativistic formulation for the inclusion of charge in the TOV equation and present the numerical procedure to solve them for a given radial charge distribution inside the star. Section III presents the application of these equations to polytropic stars wherein different aspects of the effect of charge are described, such as mass-radius-charge diagrams, the inside electric fields and the metric. In section IV, we discuss the stability of these stars and the possibility of formation of charged black holes. Finally we draw our conclusions in section V.

II. GENERAL RELATIVISTIC FORMULATION

We take the metric for our static spherical star as

$$ds^2 = e^\nu c^2 dt^2 - e^\lambda dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (1)$$

The stress tensor T_ν^μ will include the terms from the Maxwell's equation and the complete form of the Einstein-Maxwell stress tensor will be :

$$T_\nu^\mu = (P+\epsilon)u^\mu u_\nu - P\delta_\nu^\mu + \frac{1}{4\pi}\left(F^{\mu\alpha}F_{\alpha\nu} - \frac{1}{4}\delta_\nu^\mu F_{\alpha\beta}F^{\alpha\beta}\right) \quad (2)$$

where P is the pressure, ϵ is the energy density ($=\rho c^2$) and u^μ is 4-velocity vector. For the time component, one easily sees that $u^t = e^{-\nu/2}/c$ and hence $u^t u_t = 1$. Consequently, the other components (radial and spherical) of the four vector are absent.

Now, the electromagnetic field is taken from the Maxwell's field equations and hence they will follow the relation

$$\left[\sqrt{-g}F^{\mu\nu}\right]_{,\nu} = 4\pi j^\mu\sqrt{-g} \quad (3)$$

where j^μ is the four-current density. Since the present choice of the electromagnetic field is only due to charge, we have

only $F^{01} = -F^{10}$, and the other terms are absent. In general, we can derive the electromagnetic field tensor $F^{\mu\nu}$ from the four-potential A_μ. So, for non vanishing field tensor, the surviving potential is $A_0 = \phi$. We also considered that the potential has a spherical symmetry, i.e., $\phi = \phi(r)$.

The non vanishing term in Eq. (3) is when $\nu = r$. This gives the electric field as

$$\begin{matrix} t \\ t \end{matrix} \sim \begin{matrix} r \\ r \end{matrix} : \quad \frac{1}{4\pi}\left(F^{\mu\alpha}F_{\alpha\nu} - \frac{1}{4}\delta^\mu_\nu F_{\alpha\beta}F^{\alpha\beta}\right) = \frac{-\mathcal{U}^2}{8\pi}$$

where,

$$\mathcal{U}(r) = \frac{1}{r^2}\int_0^r 4\pi r^2 j^0 e^{(\nu+\lambda)/2}dr \qquad (4)$$

is the electric field. Let us consider a distribution of charge in the matter as some charge density annoted as $\rho_{\rm ch}$. We modify the above expression (Eq. (4)) and rewrite it as

$$\mathcal{U}(r) = \frac{1}{r^2}\int_0^r 4\pi r^2 \rho_{\rm ch} e^{\lambda/2}dr. \qquad (5)$$

This leads to defining the total charge of the system as

$$Q = \int_0^R 4\pi r^2 \rho_{\rm ch} e^{\lambda/2}dr \qquad (6)$$

where R is the radius of the star.

With the metric (1) one can easily get the Einstein's field equations from the relation

$$R^\mu_\nu - \frac{1}{2}R\delta^\mu_\nu = -\frac{8\pi G}{c^4}T^\mu_\nu$$

as

$$\begin{matrix} t \\ t \end{matrix} : \quad \frac{e^{-\lambda}}{r^2}\left(r\frac{d\lambda}{dr} - 1\right) + \frac{1}{r^2} = \frac{8\pi G}{c^4}\left(\epsilon + \frac{\mathcal{U}^2}{8\pi}\right) \qquad (7)$$

$$\begin{matrix} r \\ r \end{matrix} : \quad \frac{e^{-\lambda}}{r^2}\left(r\frac{d\nu}{dr} + 1\right) - \frac{1}{r^2} = \frac{8\pi G}{c^4}\left(P - \frac{\mathcal{U}^2}{8\pi}\right). \qquad (8)$$

The first of the Einstein's equations is used to determine the metric e^λ. The mass of the star is now due to the total contribution of the energy density of the matter and the electric energy ($\frac{\mathcal{U}^2}{8\pi}$) density. The mass takes the new form as

$$M_{\rm tot}(r) = \int_0^r 4\pi r^2 \left(\frac{\epsilon}{c^2} + \frac{\mathcal{U}^2}{8\pi c^2}\right)dr \qquad (9)$$

and the metric coefficient is given by

$$e^{-\lambda} = 1 - \frac{2GM_{\rm tot}(r)}{c^2 r}. \qquad (10)$$

However, this mass is the one measured in the star's frame. For an observer at infinity, the mass [5, 16] is given by

$$\begin{aligned} M_\infty &= \int_0^\infty 4\pi r^2 \left(\frac{\epsilon}{c^2} + \frac{\mathcal{U}^2}{8\pi c^2}\right)dr \\ &= \int_0^R 4\pi r^2 \left(\frac{\epsilon}{c^2} + \frac{\mathcal{U}^2}{8\pi c^2}\right)dr + \\ &\quad \int_R^\infty 4\pi r^2 \left(\frac{\epsilon}{c^2} + \frac{\mathcal{U}^2}{8\pi c^2}\right)dr \\ &= M_{\rm tot}(R) + \frac{Q(R)^2}{2R} \end{aligned} \qquad (11)$$

where R is the radius of the star. In our plots and rest of the text, we will refer to the mass M as the $M_{\rm tot}$.

From the conservation of stress tensor ($T_\nu{}^\mu{}_{;\mu} = 0$) one gets the form of the hydrostatic equation. To this end, we obtain the modified TOV as

$$\frac{dP}{dr} = -\frac{G\left[M_{\rm tot}(r) + 4\pi r^3 \left(\frac{P}{c^2} - \frac{\mathcal{U}^2}{8\pi c^2}\right)\right](\epsilon + P)}{c^2 r^2 \left(1 - \frac{2GM_{\rm tot}}{c^2 r}\right)} \\ + \rho_{\rm ch}\,\mathcal{U}\,e^{\frac{\lambda}{2}}. \qquad (12)$$

The first term on the r.h.s. comes form the gravitational force with an effective pressure and density which we will discuss later, and the second term from the Coulomb force that depends on the matter by the metric coefficient.

Numerical solutions in fortran require the integral forms of Eqs. (5), (9) & (10) to be expressed in their differential forms. We can write the corresponding differential forms as

$$d\mathcal{U} = -\frac{2\mathcal{U}dr}{r} + 4\pi\rho_{\rm ch}e^{\lambda/2}dr, \qquad (13)$$

$$dM_{\rm tot} = 4\pi r^2 \left(\frac{\epsilon}{c^2} + \frac{\mathcal{U}^2}{8\pi c^2}\right)dr \qquad (14)$$

and

$$d\lambda = \left[\frac{8\pi G}{c^2}re^\lambda\left(\frac{\epsilon}{c^2} + \frac{\mathcal{U}^2}{8\pi c^2}\right) - \left(\frac{e^\lambda - 1}{r}\right)\right]dr. \qquad (15)$$

So, the final four equations needed to be solved are Eqs. (12), (13), (14) & (15). The boundary conditions for the solution are, at the centre where $r = 0$, $\mathcal{U}(r) = 0$, $e^{\lambda(r)} = 1$, $P(r) = P_c$ and $\rho(r) = \rho_c$, and at the surface where $r = R$, $P(r) = 0$. The inputs in the equations are the pressure P, the energy density ϵ and the charge density $\rho_{\rm ch}$. The metric coefficient λ and the electric field $\mathcal{U}$ are interdependent. This gives us a set of four coupled differential equations which we solve simultaneously to get our results. We note here that the form of the equations does not change with the sign of the charge because the electric field appears in the mass term (14) and pressure gradient term (12) in squares and in the Coulomb part, the product $\rho_{\rm ch}\mathcal{U}$ is also invariant.

III. CHARGED STARS IN A POLYTROPIC EQUATION OF STATE

A. The Mass-Radius relation and other features

We now examine the effects of charge in a polytropic equation of state (EOS), which is a more general approach than considering any model dependent EOS. We make the charge go with the mass density (ϵ) as

$$\rho_{\rm ch} = f \times \epsilon \qquad (16)$$

where $\epsilon = \rho c^2$ is in [MeV/fm^3]. With this assumption, the charge fraction f has a dimension $\frac{1}{[\rm km]}[\rm fm^3/MeV]^{1/2}$ and

the charge density $\rho_{\rm ch}$ is in $[{\rm MeV/fm}^3]^{1/2}\frac{1}{[{\rm km}]}$. This kind of 'mixed units' appear in our dimensions of f and $\rho_{\rm ch}$ because one can see from Eqs. (5) & (12) that the electric field is proportional to the square root of the pressure (in units of MeV/fm^3) and the integration over the radius r is carried out in kilometers. In geometrical units, this can be written as

$$\rho_{\rm ch} = \alpha \times \rho \qquad (17)$$

where charge is expressed in units of mass and charge density in units of mass density. This α is related to our charge fraction f as

$$\alpha = f \times \frac{0.224536}{\sqrt{G}}$$
$$= f \times 0.86924 \times 10^3. \qquad (18)$$

Our choice of charge distribution is a reasonable assumption in the sense that large mass can hold a large amount of charge.

The polytropic EOS is given by

$$P = \kappa \rho^{1+1/n} \qquad (19)$$

where n is the polytropic index and is related to the exponent Γ as $n = \frac{1}{\Gamma - 1}$. In the relativistic regime, the allowed value of Γ is $\frac{4}{3}$ to $\frac{5}{3}$. We have considered the case of $\Gamma = \frac{5}{3}$ and the corresponding value of n is 1.5. Primarily, our units of matter density and pressure are in MeV/fm^3. We choose a value of κ as 0.05 $[fm]^{8/3}$. Thus we have an equation of state which we analyze for different cases of charge fraction f and study the nature and behavior of the system. The choice of the EOS is however not very important in so far as the nature of the curves due to the effect of charge is concerned, and show only corresponding shift in the maximum mass based on the type of EOS used.

It should be noted that the nuclear forces are only affected by electromagnetic forces (and so the EOS) when the number density of the charged particles are of the order of the baryonic number density, i.e., Z $\sim$ A. This has been verified previously in many works on the nuclear structure. However, for the case of charged stars, the forces are compared between the electric and the gravitational force. For our case, the Z/A ratio is $\sim 10^{-18}$, which essentially means that these extra charged particles which produces huge electric field and affects the structure of the star, produces negligible effect on the nuclear matter and the EOS. The same argument holds for the chemical potential, which are controlled by chemical equilibrium and charge neutrality. The extra charged particles are just very few when compared to the total number of baryons of the star. Any effect brought in by these extra charged particles can be approximated (and compared) to the case of chemical equilibrium and zero charge. So, it is justified to use an EOS which is calculated for neutral matter. We have indeed tested the effect of this amount on charge on the EOS on some sophisticated models of nuclear matter like the Non-linear Walecka model [17, 18], and verified our reasoning to be right.

We plot the mass as a function of the central density ρ_c in Fig. (1) for different values of the charge fraction f. The stars which are on the higher density regime and have lower mass,
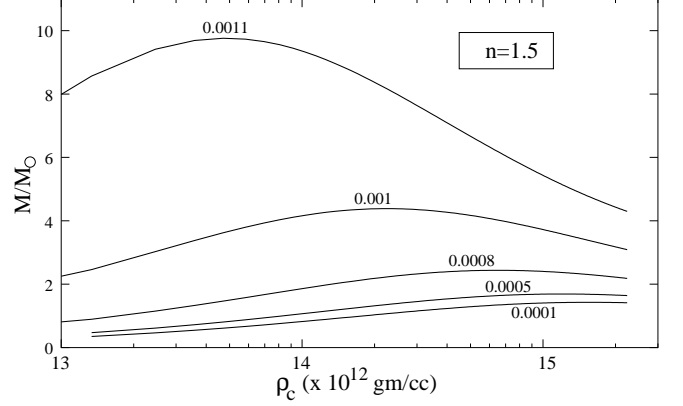

FIG. 1: Central density against mass for different values of the factor f.

are unstable because $\frac{dM}{d\rho_c} < 0$. Those falling in the lower density regime and have increasing mass are all allowed. The effect of the charge for $f = 0.0001$ on the structure of the star is not profound and is comparable with that of chargeless star. This value of f is however 'critical' in the sense that further increase in the value shows effect on the structure of the star. With the increase of the charge fraction form $f = 0.0001$ to $f = 0.0005$ the structure changes by 20% increase in the value, from $f = 0.0005$ to $f = 0.0008$, the increase is 35% and from $f = 0.0008$ to $f = 0.001$, the change is almost 90%, thus showing that the change in the structure is non-linear with the change in the charge fraction as can be seen in Fig. (1).
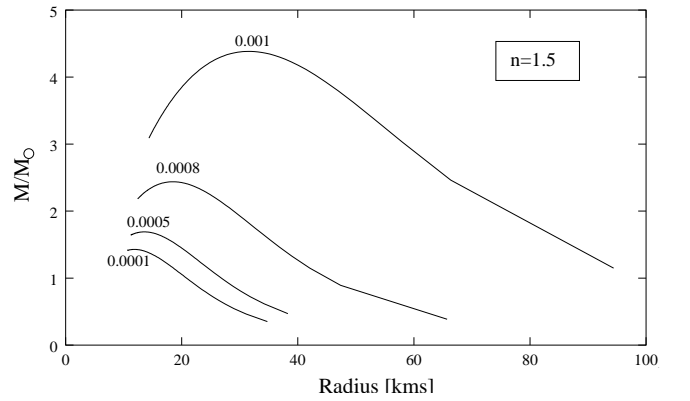

FIG. 2: Mass-Radius relation for different values of the factor f.

In Fig. (2) we plot the mass-radius relation. Due to the effect of the repulsive force, the charged stars have large radius and larger mass as we should expect. Even if the radius is increasing with the mass, the M/R ratio is also increasing, but much slower. For the lower charge fractions, this increase in the radius is very small, but a look at the structure for the fraction $f = 0.001$ reveals that for a mass of 4.3 M$_\odot$, the radius goes as high as 35 km. Though the compactness of the stars are retained, they are now better to be called as 'compact

charged stars' rather than 'charged neutron stars'. The charge fraction in the limiting case of maximally allowed value goes up to $f = 0.0011$, for which the maximum mass stable star forms at a lower central density even smaller than the nuclear matter density. This extreme case is not shown in Fig. (2) because the radius of the star and its mass is very high (68 km and 9.7 $M_\odot$ respectively). These effects suppress the curves of the lower charge fractions due to scaling. For this star, the mass contribution from the electric energy density is 10% than that from the mass density. It can be checked by using relation (18) that this charge fraction $f = 0.0011$ corresponds to $\rho_{\rm ch} = 0.95616 \times \rho$ in geometrical units. In Table I, we show the maximum mass for different charge fractions and their corresponding radii, central densities and net charge content.

TABLE I: The maximum allowed stable stellar configuration for different charge fractions.

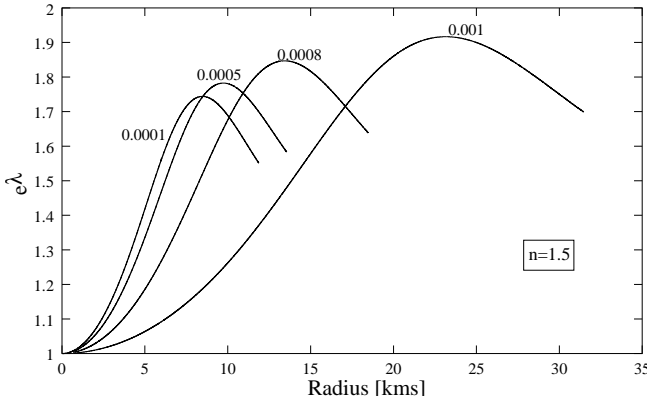| f	$M_{\rm tot}$ ($M_\odot$)	M_∞ ($M_\odot$)	R (km)	ϵ_c (MeV/fm^3)	Q ($\times 10^{20}$ Coulomb)
0.0001	1.428	1.43	11.87	1550.41	0.259
0.0005	1.69	1.765	13.55	1202.6	1.517
0.0008	2.438	2.728	18.47	652.87	3.434
0.001	4.384	5.248	31.47	226.55	7.576
0.0011	9.76	12.15	68.7	47.04	18.314

FIG. 3: Variation of the metric e^λ with the radius in the maximum mass stars for different cases of charge distribution with varying f.

In Fig. (3), we plot the metric coefficient e^λ as a function of radius for the maximum mass stars for each of the charge fractions. A quick comparison shows that the nature of e^λ is the same for all the stars with different charges. There is a slight increase in its value for higher charge fractions thus showing the gain in the compactness $\frac{M}{R}$ of the star with charge. This can be verified from the values of e^λ at the surface for two cases of charge fractions $f = 0.0001$ and $f = 0.001$, $\left(\frac{2M}{R}\right)_{0.0001} = 1 - \frac{1}{1.55} = 0.3548$, and $\left(\frac{2M}{R}\right)_{0.001} = 1 - \frac{1}{1.7} = 0.4118$. This re-confirms that the compactness of the star increases despite the enormous increase in the radius with the increase of the charge fraction.

It is interesting to point out that we have tested our model of charged stars for a very soft EOS which can go upto a very high density, where we saw that the spiraling behaviour of the mass-radius curve exists for high charges. This nature of the curve are very much supportive to the fact that these charged stars are stable to stellar oscillations.

B. Charge and Electric field inside the star.

In this subsection we discuss the effect of the charge in the inner profile of the star. In the $Q \times R$ diagram in Fig. (4), we plot the total charge Q of the stars at the surface as function of their radius. From Eq. (6), it is clear that the charge will increase with the increasing charge fraction. This increase is however not the only one responsible for the high charges if we consider that equation. There are contributions also from the large radius of the higher charge configuration and the larger value of the metric coefficient e^λ. So, we see that the matter and charge are inter-related to each other and the effect of one depends intrinsically on the other and hence the relations are strongly coupled.
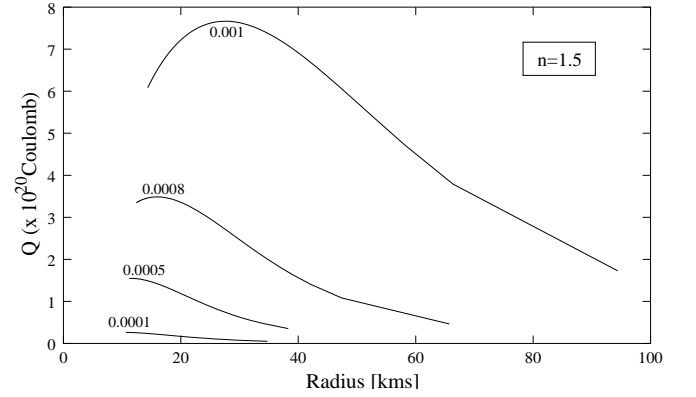

FIG. 4: The variation of the charge with radius for different f.

The charge developed due to this range of charge fraction ($f = 0.0001$ to $f = 0.0011$) is in the limiting case where the effect is noticed on the structure. This scale of the charge is easy to understand from the mass expression and the modified TOV (Eqs. (9) and (12)) where in order to see any change in the stellar configuration, we should have

$$\mathcal{U} \simeq \sqrt{8\pi P} < \sqrt{8\pi\epsilon}. \tag{20}$$

If we consider that $P \sim 10$ MeV/fm^3, then $\mathcal{U} \simeq 10\left(\frac{\rm MeV}{\rm fm^3}\right)^{1/2}$ and we show in appendix that with proper conversion, this gives $\mathcal{U} \simeq 10^{22}$V/m. With R$\simeq$10 km, the charge needs to be at the order of 10^{20} Coulombs.

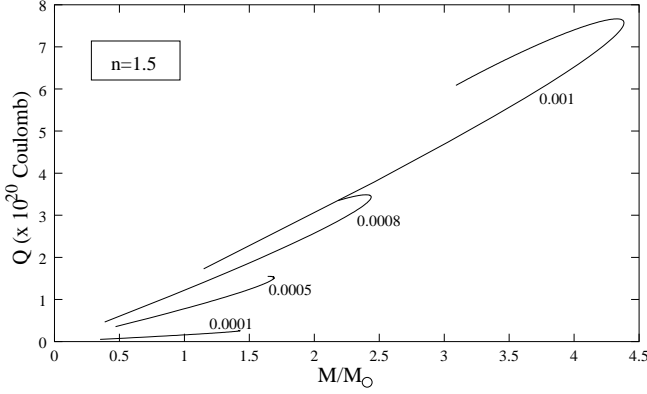In the $Q \times M$ diagram in Fig. (5), we plot the mass of the stars against their surface charge. We have made the

FIG. 5: The variation of the charge with mass for different f.

FIG. 6: Variation of $\mathcal{U}$ with the radius in the maximum mass stars for different cases of charge distribution with varying f.

charge density proportional to the energy density and so it was expected that the charge, which is a volume integral of the charge density, will go in the same way as the mass, which is also a volume integral over the mass density. The slope of the curves comes from the different charge fractions.The nature of the curves in fact reflect that charge varies with mass (with the turning back of the curves all falling in the 'unstable zone' and is not taken into consideration). If we consider that the maximum charge allowed is estimated by the condition (20) for $\frac{dP}{dr}$ to be negative (Eq. (12)), we see that the curve for the maximum charge in Fig. (5) has a slope of 1:1 (in a charge scale of 10^{20} Coulomb). This scale can be easily understood if we write the charge as

$$Q = \sqrt{G} M_\odot \frac{M}{M_\odot} \simeq 10^{20} \frac{M}{M_\odot} \text{ Coulomb.} \qquad (21)$$

It is worth mentioning that this charge Q is the charge at the surface of the star where the pressure and also $\frac{dP}{dr}$ are already very small (ideally zero). So, at the surface, the Coulomb force is essentially balanced by the gravitational force and the relation of the charge and mass distribution we found is of the same order (i.e., $Q \simeq M$ or $\rho_{ch} \simeq \rho$) for the case of charged dust sphere discussed earlier by Papapetrou [12] and Bonnor [13], as we referred in the introduction.

In Fig. (6), the electric field ($\mathcal{U}$) is plotted as a function of the stellar radius for the maximum mass star for different charge fraction f. The radius is shown in a log-scale. From the expression (Eq. (5)) of the electric field, it is clear that the profile of the field will depend on the charge fraction f, the metric coefficient e^λ and also the radius of the star. The value of the field increases for charge fraction up to $f = 0.0008$ but then falls down. The decrease in the field for the higher charge fraction is attributed to the formation of the stable star at a very small density (Fig. (1)) for that particular charge fraction f.
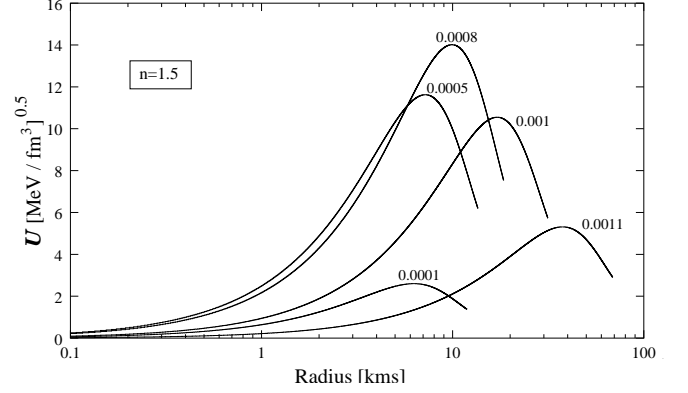
C. The modified EOS inside the stars

¿From Eqs. (9) and (12), the effective energy density and pressure of the system can be viewed as

$$P^* = P - \frac{\mathcal{U}^2}{8\pi} \qquad (22)$$

and

$$\epsilon^* = \epsilon + \frac{\mathcal{U}^2}{8\pi}. \qquad (23)$$

In Figs. (7) and (8) we show two different charge fractions $f = 0.0005$ and $f = 0.0008$, the effective pressure (P^*) and energy density (ϵ^*). The effective pressure drops down to a negative value, but even in this case, the first part of Eq. (12) preserves its overall negative value because ($M_{tot} + 4\pi r^3 P^*$) is positive. Thus the overall sign of the pressure gradient $\frac{dP}{dr}$ is still negative as long as the attractive gravitational term is larger than the repulsive Coulombian one. These figures show that the effective EOS becomes stiffer due to the inclusion of charge and consequently allowing more mass in the star.

Also, the effective pressure directly reduces the value of the negative part of $\frac{dP}{dr}$, but the effective energy density increases the same through the M_{tot}. This goes on until the effective pressure becomes so much negative that it overcomes the value of the M_{tot} and this limits the formation of star with higher charge fraction f.

D. Balance between the Coulomb and gravitational forces

In this section, we discuss the effects brought in the pressure gradient from the matter energy and the Coulomb energy. As mentioned previously, the total mass of the system M_{tot} increases with increasing charge because the electric energy density 'adds on' to the mass energy density. This change in the mass is low for smaller charge fraction and going up to seven times the value of chargeless case for maximum allowed charge fraction $f = 0.0011$. This effect however does not
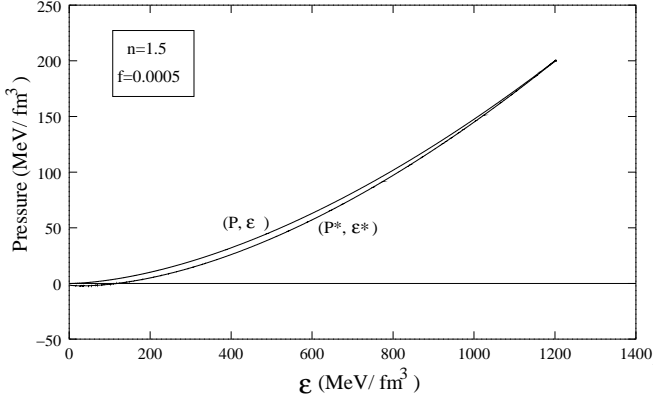
FIG. 7: The pressure versus energy density plot for chargeless case and for charge fraction $f = 0.0005$.
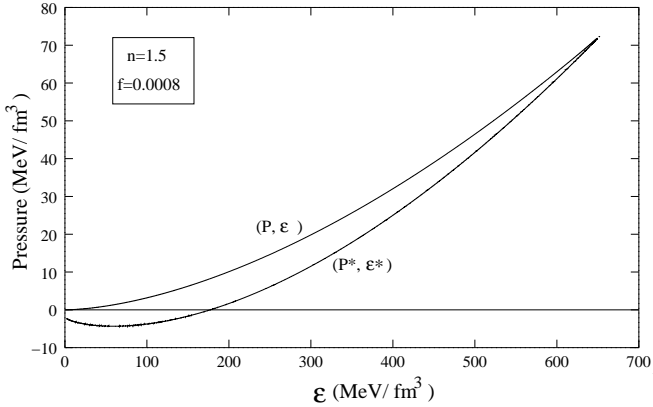

FIG. 8: The pressure versus energy density plot for chargeless case and for charge fraction $f = 0.0008$.

change the metric coefficient considerably when compared to the chargeless case (see Fig. (3)). So, the ruling term in Eq. (12) is the factor $(M_{\text{tot}} + 4\pi r^3(P - \frac{U^2}{8\pi}))$. With the increase of charge, the value of $(P - \frac{U^2}{8\pi})$ decreases, and hence the gravitational negative part of Eq. (12) decreases. The central pressure is very high. So, with the softening of the pressure gradient, the system allows more radius for the star until it reaches the surface where the pressure (and the pressure gradient too) goes to zero. We should stress that because $\frac{U^2}{8\pi}$ cannot be too much larger than the pressure in order to maintain $\frac{dP}{dr}$ negative as discussed before, so we have a limit on the charge. It is interesting that this limit comes from the relativistic effects of the gravitational force and not just only from the repulsive Coulombian part.

This effect is illustrated in Fig. (9) where we plot both the positive Coulomb part and the negative matter part of the pressure gradient. Note that, although we mention a Coulomb
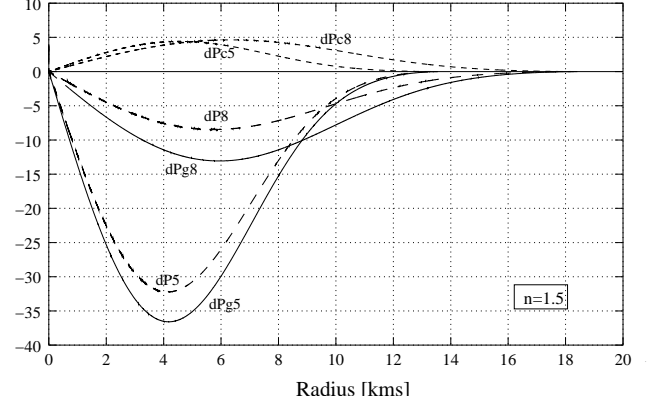

FIG. 9: The Pressure gradient with the positive Coulomb part and the gravitational negative part is shown here for two different values of the charge factor f (=0.0005 and 0.0008). The total, $\frac{dP}{dr}$, is also plotted for the same. The annotations for the component of pressure for two charges $f = 0.0005$ and 0.0008 coming from the *matter part* are dPg5 and dPg8 respectively, those from *Coulomb part* are dPc5 and dPc8 respectively and the corresponding totals are dP5 and dP8.

part and a matter part, both are coupled with each other in the sense that charge changes the mass and the mass changes the Coulomb part. The plots are for two values of the charge fraction $f = 0.0005$ and $f = 0.0008$. The positive part of $\frac{dP}{dr}$ maintains its almost constant value because the charge fraction f is the controller of the same, and in our case, they differ by a very small percentage. In the negative part, the changes are drastic and are mainly brought by the effective pressure as we already discussed.

IV. STABILITY OF CHARGED STARS AND FORMATION OF CHARGED BLACK HOLES

A. Stability considerations

Here we will discuss the stability of these charged compact stars taking into account the forces acting only on the charged particles. For this, we need to compare our results with the stable configuration of an almost neutral neutron star.

The basic argument to assume the charge neutrality of Newtonian stars, or quasi-Newtonian stars (such as neutron stars), is based on the fact that the total charge of the stars should lie below a certain limit where the Coulomb repulsive force overwhelms the gravitational attractive force at the surface of the star. This limiting value can be viewed as the Coulomb force acting on a proton at the surface and we have the limiting range as :

$$\frac{(Ze)e}{R^2} \leq \frac{GMm}{R^2} \leq \frac{G(Am)m}{R^2} \qquad (24)$$

where, Ze, R and M are the net charge of the star, its radius and mass respectively and m and e are the mass and charge

of a proton. In the above equation, the mass of the star is considered to be smaller than Am because of the gravitational binding of the system. In gravitational units,

$$\frac{Z}{A} < \left(\frac{m}{e}\right)^2 < 10^{-36}. \qquad (25)$$

If we take into account that there are approximately $A \simeq 10^{57}$ baryons in a neutron star ($M \simeq M_{\odot}$), so $Z < 10^{21}$, which gives

$$Q \simeq Ze \simeq 100 \text{ Coulomb}. \qquad (26)$$

This is the limit on the net positive charge already discussed in the introduction. If the star had a net negative charge, then the electrons being the carrier of charge, the value of the limit would be reduced by the factor m_e/m [4]. Using the solution for the TOV equation in the presence of electric fields, which we already solved in the previous sections, we showed that the balance of the forces, to make an element of the fluid at rest (hydrostatic equilibrium equation) allows a very large amount of charge.

In fact we showed that the maximum allowed charge is obtained when $\rho_{\text{ch}} \simeq \sqrt{G}\rho$ in natural units from the assumption that more mass can hold more charge ($\rho_{\text{ch}} \propto \rho$). Because of the large density found in the compact stars like neutron stars ($\rho \simeq 10^{15}$ g/cm^3) we can expect a large charge if ρ_{ch} is of the same order as ρ. Thus, $Q(r) \simeq \sqrt{G}M(r)$ and at the surface of a neutron star with $M \simeq M_{\odot}$, the total charge is $Q \simeq \sqrt{G}M_{\odot} \simeq 10^{20}$ Coulomb and the intensity of the electric field is of the order of $10^{21} - 10^{22}$ V/m. In the case of maximum charge, the relation $Q(r) = \sqrt{G}M(r)$ at the surface shows that the ratio $Z/A \simeq 10^{-18}$ as compared to 10^{-36} in Eq. (25). This explains why we are having charges 10^{18} times larger than that in relation (26). This Z is the net charge in the star and is the difference of the charged particles with opposite sign ($Z \sim Z_{net} \sim |Z_+ - Z_-|$). The number of charges of equal and opposite sign is not at all limited.

However, not only a global balance of all the forces is enough to guarantee the stability of these stars when we consider the forces acting on each of the charged particles of that fluid. As the Z/A ratio is $\simeq 10^{-18}$, the Coulomb force is 10^{18} times larger than the gravitational one felt by a charged particle. This will make the charged particles leave the system, making the star unstable unless any other mechanism exists to bind together one charged particle with 10^{18} neutral particles.

Our analysis reveals that the maximum amount of charge which can be allowed in the system of charged compact stars is close to unity (precisely 0.95) in geometrical units and can be compared to the extremum case of charge in a Reissner-Nordström black hole, i.e., $Q/\sqrt{G}M = 1$. The stability analysis of the charged star from the viewpoint of the force acting on a single charged particle makes the star highly unstable, with the charge escaping from the star within a very short time. However, very little amount of charge will escape before it collapses to a charged black hole. This can be seen as follows : with the escape of little amount of charge, the repulsive pressure from the Coulomb part diminishes, but the attractive pressure from the matter part is not affected very

much. This is because the gravitational contribution of the escaped charged particle (say proton) is practically zero as compared to that from the 10^{18} neutral particles. This leads to a very critical scenario of disbalance of the global forces inside the star, where the loss of a little amount of charge makes the gravitational attractive pressure overwhelm the residual repulsive Coulombian pressure and the whole system collapses further to form black holes. But all the charge has not yet escaped from the system and so, the residual charge gets trapped in the black hole forming the charged black holes. This picture is best suitable for the extreme charged case, but can be true for smaller charged fractions also. We saw in Fig. (1), that the maximum stable mass for the highest allowed charge fraction ($f = 0.0011$) is as high as 9.7 $M_{\odot}$ and such high mass charged compact stars are more favorable to collapse to Reissner-Nordström black holes.

B. Discharge time

Since the electric force on a charged particle is much greater than the gravitational force one can neglect the gravitational force, and find that, from Newtonian theory, the equation of motion for a charged particle is

$$m \frac{\partial^2 r}{\partial t^2} = \frac{Q q}{r^2}$$

where m and q are the mass and charge of the particle, and Q is the net charge of the star. Thus, the lifetime to discharge the star $t_{\text{discharge}}$ is given in Newtonian theory by $t_{\text{discharge Newt}} \simeq \frac{1}{c}\sqrt{\frac{R^3}{GM/c^2}\frac{m}{q}}$, where we have used the relation $Q \simeq \sqrt{G}M$. For $R \sim 10$ km, $GM/c^2 \sim 3$ km and $m/q \sim 10^{-18}$, we have, $t \sim 10^{-13}$ s. However this time scale is much shorter than the time a particle takes to traverse the radius of the star. This means that the charged particle acquires quickly a velocity near the speed of light and it will travel throughout the star with this speed. Thus, the discharging time is of the order of the crossing time, i.e.,

$$t_{\text{discharge}} \simeq \frac{R}{c}$$

For a star with $R \sim 10$ km $t_{\text{discharge}} \simeq 10^{-5}$ s.

We have not taken into account that the particle collides with other particles with a mean free path of the order of one fermi. However, the trajectory of the charged particle is not going to be a random walk trajectory, since the electric force is radial. Thus, this collision process might increase a bit the discharging time, but not a lot. A detailed evaluation is however a very complicated process and is beyond the scope of the present paper.

V. CONCLUSIONS

Our analysis shows that the amount of charge contained in a dense system like a compact star can be very high and several orders of magnitude larger than those calculated by classical balance of forces at the surface of a star. This amount

of charge mainly comes from the high density of the system since $\rho_{ch} \propto \rho$. We showed that the charge can be as high as 10^{20} Coulomb to bring in any change in the mass-radius relation of the star, yet remaining stable as long as one considers only the hydrostatic equilibrium and the global balance of forces and not considering the individual forces on the charged particles. In our study, we used a polytropic EOS for our compact star with a choice of parameters such that the system is close to the realistic neutron stars. We showed that in the critical limit of the charge contained in the system, the maximum mass stable star forms in a lower density regime, however compactness keeps on increasing. We have also studied the change in the pressure gradient due to the effect of the charge contained in the system. It was expected from the classical picture of forces involved, that the repulsive force of the charged particles will add up to the internal pressure of the system and the entire repulsive force will be balanced by the gravitational force of the system. However, we can see from Fig. (9) that the contributions from the charged particles are helping to soften the gravity part of the pressure gradient, thus allowing more matter to stay in the system. The second term in the right hand side of Eq. (12) is positive always and does not depend on the nature of the charge whether it is positive or negative. The pressure gradient must be a negative term and hence $\frac{dP}{dr}$ is softened by the effect of the presence of charge. The net effect is the gravitational force which tries to collapse the system is held further away by the Coulomb force, which, in the absence of the gravity would have exploded the system.

This imbalance of the pressure actually will come to play when some of the charge will leave the star due to the Coulombian repulsion acting on a single charged particle. As we discussed in section IV, the attractive gravitational pressure will then become more than the repulsive total kinetic and Coulombian pressure, as a result of which the star may collapse to a charged black hole.

We have studied the electric field inside the maximum mass star allowed by a certain charge fraction. We found that the field attains a maximum value for certain amount of charge fraction and then decreases for higher charge fractions (Fig. (6)). This is also interesting because normally it was expected that the increase in the charge fraction would increase the field also. But as our charge distribution varies directly with the matter density, so the formation of the maximum mass star in the lower density regime for high charge fractions, reduces the electric field. The mass of the stars (Eq. (9)) in these critical field limit is very high as compared to the chargeless case and have a contribution from the electrostatic energy density, but this is small compared to the contribution of the matter density.

As pointed by Bekenstein [5], with this amount of huge charge (as we have in our system), the electric field produced will be too high and give rise to pair production. This effect results in self-diminishing the electric field and thus the system will destabilize [19]. However, we can say that the critical field limit calculation has been done in vacuum, and it is not at all clear how the field will behave in a dense system. Recent observations have revealed that there are magnetars, stars with strong rotating fields, which have a magnetic field as high as 10^{15} Gauss. The critical magnetic field limit for pair creation in vacuum is 10^{13} Gauss. So, if such high fields really exist in a highly dense magnetar and these stars are stable, then the critical field limit needs to be modified for high density matter. Putting aside this debate of the critical field limit, we have checked only the behaviour of the system in the presence of a very high charge. We have found that the global balance of forces between the matter part and the electrostatic part can support a huge amount of charge. In addition, we have argued that the solution is unstable, and a decrease in the electric field can create an enormous imbalance, resulting in the collapse of the charged star to a charged black hole. Finally, these charged stars should be very short lived, they would exist within the very short period between the supernova explosion and the formation of the charged black holes.

APPENDIX: CONVERSION TO REAL UNITS

It is important to mention here the basic units used in our approach and their conversions from these units to the real units of the charge and fields like Coulomb and Volt/meter respectively. We used the charge density (ρ_{ch}) proportional to the energy density of the system (ρ), which in turn is in MeV/fm^3, with a factor f which has dimensional units. From the fine structure constant $\alpha = \frac{e^2}{\hbar c} = \frac{1}{137}$, we get a relation $1\,\text{statCoulomb}(1\,\text{e.s.u.}) = 2.51 \times 10^9\,[\text{MeV fm}]^{1/2}$. Using the conversion factor $1\,\text{Coulomb} = 2.998 \times 10^9\,\text{e.s.u.}$ for the charge we arrive at

$$1\,\text{Coulomb} \simeq 0.75 \times 10^{19}\,[\text{MeV fm}]^{1/2}. \qquad (A.1)$$

Also, the units of $\frac{\mathcal{U}^2}{8\pi}$ have to be the same as the units of pressure. Initially, we have that the units of pressure are [MeV/fm^3]. This indicates that $\mathcal{U}$ will have [MeV/fm^3]$^{1/2}$ units. Working out the relations, we find

$$1\left[\frac{\text{MeV}}{\text{fm}^3}\right]^{1/2} = 1.2 \times 10^{21}\,\text{V/m}. \qquad (A.2)$$

Also,

$$1\left[\frac{\text{MeV}}{c^2\text{fm}^3}\right] = 1.78 \times 10^{12}\,\text{g/cm}^3.$$

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